Appendix:

Tell-tale Signals: A Customized Toolkit for Tracking the Economy

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Vulnerability Barometer (Ambler and Kronick 2020, Kronick and Ambler 2023a)

The data underlying the vulnerability index are defined as follows.

1) Household debt to disposable income: from Statistics Canada. Household credit is for end-of-period from the monthly series in Table 10-10-0118-01. Disposable income is from Table 36-10-0104-01.
2) Housing price to rent ratio: from the OECD, indexed to 100, available at https://data.oecd.org/price/housing-prices.htm
3) Non-financial corporate debt to GDP: from Statistics Canada. Corporate debt is for end-of-period, and is the sum of the market value of debt securities, loans, and accounts payable, all from Table 36-10-0580-01. GDP is expenditure-based, quarterly, and seasonally adjusted at annual rates and market prices from Table 36-10-0104-01.
4) Financial Institution debt to GDP: from Statistics Canada. Financial institution debt is for end-of-period for chartered banks and quasi-banks and is the sum of debt securities, loans, accounts payable, all from Table 36-10-0580-01. For GDP, see above.
5) Household debt to GDP: from Statistics Canada. See above for both household debt and GDP.
6) Debt service ratio: from Statistics Canada. Table 11-10-0065-01.

Construction of the Barometer

Duprey and Roberts (2017) choose indicator variables on the basis of their ability to predict periods of financial stress within 24 months. A minimal criterion for selection is that a variable predicts periods of financial stress better than a coin flip. The methodology also involves the selection of threshold levels for the variables (only values above the threshold levels are predictors of financial stress) and a way of weighting the different indicators in order to construct an index.

The methodology comes from the machine-learning literature. Interested readers can find more details in Duprey and Roberts, Fawcett (2006), Flach (2010), Swets et al. (2000) and van Erkel and Pattynama (1998).

We do not use all variables from Duprey and Roberts, instead, focusing on the two indicators in each of their four sectors with the best predictive power, excluding those that conflate stock and flow variables (for example debt-to-GDP ratios). Our variables are mostly deviations from one-sided trends, calculated using the same information that would be available to a regulator at a particular point in time. This exercise yields the following variables as having the best predictive power of a financial stress episode:

1) the deviation from trend of the ratio of household debt to disposable income;
2) the deviation from trend of the ratio of household debt to GDP;
3) the ratio of housing price to rent;
4) the deviation from trend of the ratio of housing price to rent;

5) the year-over-year growth rate of the ratio of non-financial corporate debt to GDP;
6) the deviation from trend of the ratio of nonfinancial corporate debt to GDP;
7) the year-over-year growth rate of the ratio of financial institution debt to GDP; and
8) the deviation from trend of the ratio of financial institution debt to GDP.

We drop the ratio of housing price to rent (though keep the one-sided deviation from trend of the ratio of housing price to rent) from the sample since (unlike for Duprey and Roberts in their cross-section of countries analysis) it has low predictive power for Canada. For the debt-service ratio, we find that the deviation from trend has robust predictive power, so we add this variable to our list.

Diffusion Index (Kronick 2016)

Given G different sectors in the economy, define the vector $Y_t = (Y_{1t} \dots Y_{Gt})$, where Y_{Gt} denotes whether the output of sector g is expanding, constant, or contracting in period t. Expanding sectors are given values of 100, constant sectors get 50, and contracting sectors get 0. An unweighted diffusion index would be given by

$$D(Y_t) \equiv \frac{1}{G}\sum_{i=1}^{G} Y_{gt}$$

for each period t.

Principal components analysis allows one to rewrite Y_t as a linear combination of mutually uncorrelated components. The first principal component is the linear combination which explains the largest fraction of the variation in the data. It can be rewritten as

$$Y_t = w * w'Y_t + e_t \quad \text{or} \quad Y_t = w * B_t + e_t$$

where w is a vector representing the weights corresponding to the first component generated by estimating principal components, w' is the transpose of w, B_t represents the business cycle shock, and e_t are the idiosyncratic shocks. If we take the expected value of the right-hand side in period t, where we assume the expected value of the error term is zero, we get

$$E(Y_t) = \frac{1}{G}\sum_{i=1}^{G} Y_{gt} = E(w * B_t) = \frac{1}{G}\sum_{i=1}^{G}(w * B_{gt}).$$

Therefore, since we can interpret the unweighted diffusion index as the percentage of industries in a given quarter that are expanding, we can make a similar statement regarding the interpretation of the principal components version. Specifically, the principal components diffusion index represents the predicted percentage of industries that are expanding in a given quarter, given the common component B_t. The preference for principal components analysis is due to the fact that, when estimated correctly, it filters out idiosyncratic shocks e_t such as weather, strikes, etc. and instead focuses on the true business cycle shock B_t.

Money Overhang Index (Ambler and Kronick 2022)

To analyze the long-run relationship between money growth and inflation, the data are filtered using the Hodrick-Prescott (HP) filter, which calculates a long-term or trend component τ_t by solving the following constrained minimization problem:

$$\min_{\tau_t}\left(\sum_{t=1}^{T}(y_t-\tau_t)^2+\lambda\sum_{t=2}^{T-1}\left((\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1})\right)^2\right)$$

for the data series y_t.

The Lagrange multiplier λ is treated as a parameter which influences how smooth the trend series will be. The second term penalizes abrupt changes in the growth rate of the trend component. The λ parameter governs how heavy the penalty is. When $\lambda = 0$, there is no penalty, and the solution is to set the trend component equal to the series itself. As λ approaches infinity, the trend approaches a linear trend line.

In this context, Ambler and Kronick (2022) choose a value of λ so that when a Taylor rule for the central bank's policy rate is estimated,

$$i_t = i^* + \theta_\pi(\pi_t - \pi^*) + \theta_y(y_t - y_t^*) + \varepsilon_t$$

π_t is approximately equal to the target inflation rate π^*, and output y_t is approximately equal to potential output y_t^*, leaving only the influence of the slow-moving natural rate i^* on policy.

Monetary Policy Stance Indicator (Kronick and Ambler 2023b)

The specifications of the estimated Taylor rules are described in the text. The exact specifications are as follows.[1]

1) The basic Taylor rule:

$$i_t = \bar{r} + \pi_t + \alpha(\pi_t - \pi^*) + \beta(y_t - y_t^*),$$

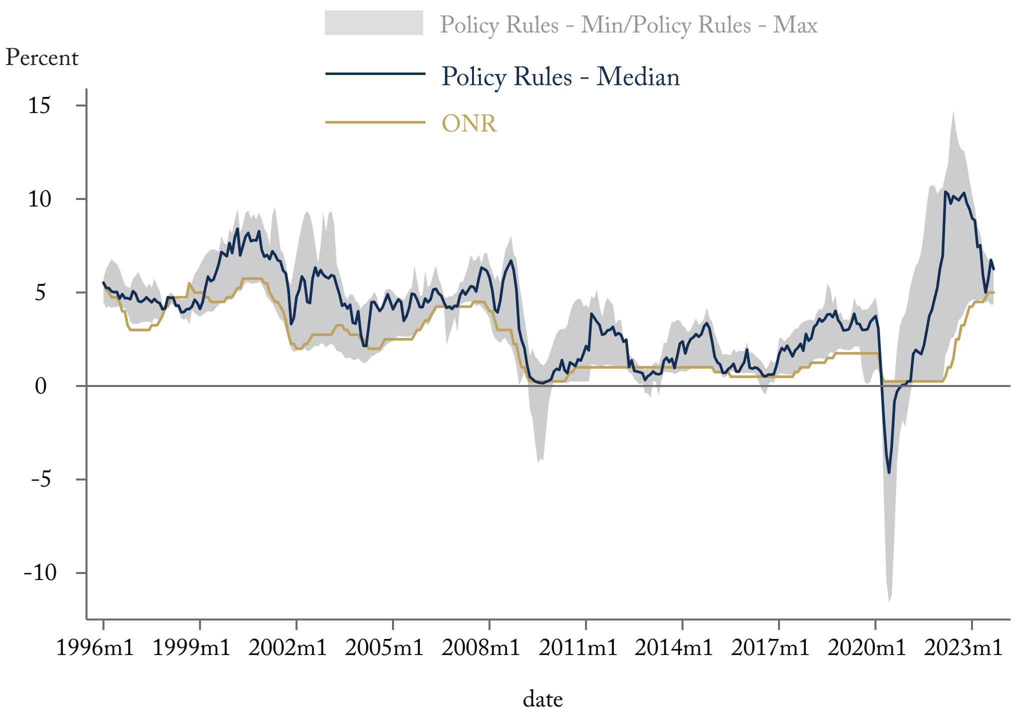
where i_t is the policy rate, $\bar{r}$ is the long-term real natural rate of interest, π_t is the current inflation rate, π^* is the inflation target, y_t is the current level of real GDP, and y_t^* is the estimated level of full-capacity GDP. As in Taylor (1993), $\alpha = \beta = 0.5$, but unlike in Taylor, we use real return bond rates for $\bar{r}$ instead of a constant real rate, and the output gap measures are from the Bank of Canada.

2) The balanced approach:

Identical to the basic Taylor rule except that the coefficient on the output gap, 1, is double that of the inflation coefficient. This is done to ensure the Bank responds more forcefully to the output gap than to inflation deviations from target.

1 For more details see Kronick and Ambler (2023b) and its online technical appendix, available at https://www.cdhowe.org/sites/default/files/2023-04/E-Brief_340%20appendix_0.pdf

Figure A1: Taylor Rule Range Indicator – Integrated Output Gap, Jan. 1996 – Sept. 2023



Source: Authors' calculations.

3) Core inflation rule:

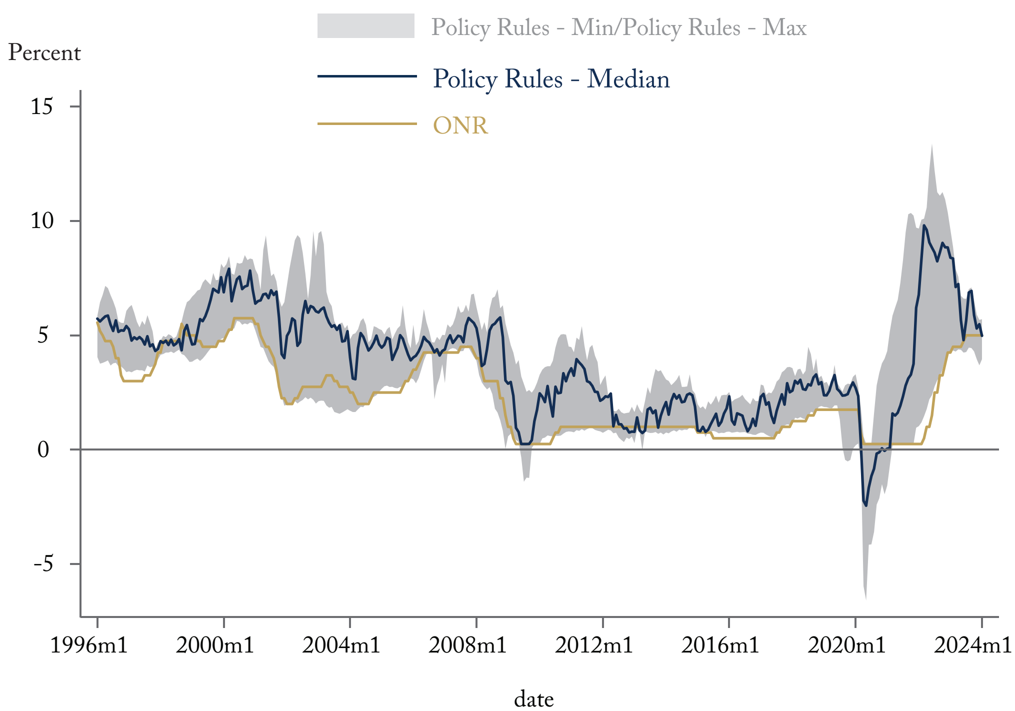
The core inflation rule is simply the balanced approach rule where headline inflation is replaced with core inflation. Core inflation is one of the Bank of Canada's preferred measures, CPI-median, which, from the Bank's website, is "a measure of core inflation corresponding to the price change located at the 50th percentile (in terms of CPI basket weights) of the distribution of price changes in a given month."

4) Forward-looking rule:

The forward-looking rule is also similar to the basic Taylor rule except that instead of using contemporaneous inflation relative to target, it uses a forecast of inflation some periods ahead. Since monetary policy works with a lag, what matters is not inflation today but inflation at some future moment in time. Kronick and Ambler (2023b) use inflation forecast data from the OECD. Following the Federal Reserve Bank of Cleveland, the formula is

$$i_t = r_t + \pi^f_{t+3} + 0.5(\pi^f_{t+3} - \pi^*) + 0.5(y_t - y^*_t).$$

Figure A2: Taylor Rule Range Indicator – Unemployment Rate, Jan. 1996 – Jan. 2024



Source: Authors' calculations.

5) Inertial rule:

   The inertial rule takes the balanced-approach rule but acknowledges that the central bank may not want to increase rates immediately, preferring to spread out the adjustments over time. The idea behind this more gradual approach is it decreases short-term interest rate volatility. The formula, from the Federal Reserve Board, is:

$$i_t = 0.85 i_{t-1} + 0.15(r_t + \pi_t + 0.5(\pi_t - \pi^*) + 1.0(y_t - y_t^*))$$

6) Effective lower bound-adjusted rule:

   An economy that requires additional stimulus may run up against the constraint that the overnight rate cannot drop any further. Taylor rules have trouble reconciling this fact, suggesting the central bank run significantly negative overnight rates when it cannot. The ELB-adjusted rule would tell the central bank to keep it at the ELB when the balanced approach suggests a negative rate. The ELB-adjusted rule would then prescribe a period of time where the overnight rate stays at the ELB despite the balanced approach suggesting it was time to raise it. In formal terms:

$$i_t = max\{i_t^{BA} - Z_t, ELB\},$$

where i_t^{BA} is the overnight rate suggested by the balanced approach rule, and Z_t represents the cumulative shortfall in monetary stimulus that occurs because short-term interest rates cannot be reduced below the ELB. This shortfall must then be shrunk to zero before the overnight rate comes off the ELB.

7) Choudhri and Schembri Taylor rule:

Choudhri and Schembri (2013) modify the simple Taylor rule (with a fixed real neutral rate) for Canada by including the US policy rate. They also use regression analysis to estimate (rather than simply choosing values thought to be approximately optimal) the values of the α and β coefficients using quarterly data. The formula is:

$$i_t = \bar{r} + \pi_t + \alpha(\pi_t - \pi^*) + \beta(y_t - y_t^*) + \gamma i_t^{US}.$$

Figures A1 and A2 above show the range of responses predicted by the different versions of the Taylor rule as well as the median value of the range for each time period. Figure A1 uses the output gap (as in Figure 4 in the main text) and Figure A2 uses the difference between the natural and actual unemployment rates (as in Figure 5 in the main text). As in Figures 4 and 5, they show that the Bank of Canada's policy rate should have started increasing above its lower bound well before it did, and that the current policy rate is consistent with the range suggested by the seven Taylor rules.[2]

Leading Economic Activity Index

For the leading economic activity index, we specifically look at 33 variables across the broad categories identified above, doing a preliminary screening on each using Granger causality tests to see if their lagged values help explain monthly real GDP growth beyond what can be explained by lagged values of real GDP growth itself (and a time trend). These tests are done after ensuring each variable is stationary (i.e., does not contain a unit root), which is true of real GDP growth, our dependent variable.[3] All variables are monthly, and growth range are taken over a three-month period on a rolling basis. The 33 variables include:

- average weekly hours, all industries, hourly workers (logs);
- average weekly hours, goods industries, hourly workers (logs);
- average weekly hours, services industries, hourly workers (logs);
- average weekly hours, all industries, salary workers (logs);
- average weekly hours, goods industries, salary workers (logs);
- average weekly hours, services industries, salary workers (logs);
- average weekly earnings, all industries, hourly workers (logs);
- average weekly earnings, goods industries, hourly workers (logs);

2 Another reason for our use of the narrower range in the main text is how problematic it is that the median interest rate was systemically above the overnight rate in the years between the GFC and pandemic, suggesting a tightening of monetary policy, despite inflation having a tough time hitting the 2 percent target from below during this period.

3 We use seasonally adjusted data in order not to have an indicator subject to seasonal fluctuations. We are aware, following Olekalns (1994) and others, that using seasonally adjusted data may reduce the power of unit root tests. In our case, first differencing the series with obvious trends (such as weekly earnings and hours) was sufficient to yield stationary variables according to our tests, which accords strongly with our prior intuition.

- average weekly earnings, services industries, hourly workers growth;
- average weekly earnings, all industries, salary workers growth;
- average weekly earnings, goods industries, salary workers (logs);
- average weekly earnings, services industries, salary workers growth;
- year-over-year growth in initial employment insurance claims;
- new orders, estimated values of orders received (manufacturers) (logs);
- total inventory, estimated values of total inventory (manufacturers) (logs);
- single dwelling permits (value) (logs);
- multiple dwelling permits (value) (logs);
- industrial dwelling permits (value) (logs);
- commercial dwelling permits (value) (logs);
- institutional and government permits (value) (logs);
- new housing price index growth;
- spread on the 2-year and 10-year Government of Canada government bonds;
- second order growth rate of M2++ broad money;
- second order growth rate of M1++ narrow money;
- Western University's Ivey Purchasing Managers Index (logs);
- Western University's Ivey Employment Index (logs);
- Western University's Ivey Inventories Index (logs);
- Western University's Ivey Suppliers Deliveries Index (logs);
- Western University's Ivey Prices Index (logs);
- TSX close (logs);
- OECD Google Tracker growth (average of all weeks in a month);
- OECD Google Tracker growth (first week of the month)[4]; and
- Bank of Canada Commodity Price Index (Total) Inflation.

Of these 32 variables, 14 were found to Granger cause real GDP growth. Our test approach involved first using the Akaike and Schwarz information criteria to determine the appropriate lag length for our dependent variable, real GDP growth. The former gave an optimal lag length of 6 while the latter gave an optimal lag length of 4. For the sake of parsimony, we went with 4. We follow Dion (1999) and test lag lengths sequentially from 6 to 1. We only discard a potential indicator if the null hypothesis that the coefficients on the lagged indicators are jointly zero cannot be rejected for each lag sequence. We differ from Dion in that we use 10 percent as the significance cut off (not 5 percent). The 14 remaining indicators include:

- average weekly earnings, all industries, hourly workers (logs);
- average weekly earnings, goods industries, hourly workers (logs);

4 We note that the OECD's Google Tracker has not published updates since April 2023. We have left it in because it does provide value in the years it is available.

- average weekly earnings, services industries, hourly workers growth;
- average weekly earnings, all industries, salary workers growth;
- average weekly earnings, services industries, salary workers growth;
- new orders, estimated values of orders received (manufacturers) (logs);
- total inventory, estimated values of total inventory (manufacturers) (logs);
- spread on the 2-year and 10-year Government of Canada government bonds;
- Western University's Ivey Purchasing Managers Index (logs);
- Western University's Ivey Prices Index (logs);
- TSX close (logs);
- OECD Google Tracker growth (average of all weeks in a month);
- OECD Google Tracker growth (first week of the month); and
- Bank of Canada Commodity Price Index (Total) Inflation.

We tried to capture only variables that provide unique information. So, for weekly earnings of hourly workers we used only the "all industries" measure – same for weekly earnings of salary workers. For manufacturers we focused on new orders. We are then left with 9 indicators for our leading economic activity index:

- average weekly earnings, all industries, hourly workers (logs);
- average weekly earnings, all industries, salary workers growth;
- new orders, estimated values of orders received (manufacturers) (logs);
- spread on the 2-year and 10-year Government of Canada government bonds;
- Western University's Ivey Purchasing Managers Index (logs);
- Western University's Ivey Prices Index (logs);
- TSX close (logs);
- OECD Google Tracker growth (first week of the month); and
- Bank of Canada Commodity Price Index (Total) Inflation.

From here we calculate the index. We follow the US Conference Board's methodology.[5] The following provides a brief summary of the five relevant steps:

- Calculate month-to-month changes for each indicator.
  - a If in interest rates, just an arithmetic difference $s_t = S_t - S_{t-1}$.
  - b All indicators for our index are in percent form except for the 2-10 spreads, where we just use the level s_t.
- Calculate the standard deviation (d) of these changes for each indicator, invert them (n), and sum them all up across indicators (call this v). Calculate the standardization factor for each indicator (f_s) by multiplying each inverted standard deviation (n) by $1/v$, so the factors sum to 1. Multiply these standardized factors (f_s) by the month-to-month change (s_t) to get the adjusted monthly contribution (c_t).
- Sum these adjusted monthly contributions, which then gives the growth rate of the index (g_t).

5 https://www.conference-board.org/data/bci/index.cfm?id=2154

- Set the first period in the sample, 1992m1, equal to 100, then calculate all subsequent months as $G_{t+1} = G_t * (200 + g_{t+1})/(200 - g_{t+1})$.[6]
- Re-base to average 100 in the base year, which is set to 2002. The index is multiplied by 100 and divided by the average of the 12 months that make up 2002.

Demand-side/Supply-side Inflation Decomposition (Chen and Tombe 2023)

Chen and Tombe's methodology involves estimating bivariate VAR (vector autoregression) models of prices and quantities in different sectors, of the form

$$Az_t = \sum_{i=1}^{p} A_i z_{t-i} + \epsilon_t,$$

where z_t is a vector of data, A_i and A are matrices of parameters, and ϵ_t are shocks.

Under certain conditions,

$$z_t - E\left[z_t | z_{t-1}, \dots, z_{t-p}\right] = A^{-1}\,\epsilon_t = v_t$$

Unexpected changes in z_t provide information about the unobservable structural shocks ϵ_t. A governs the contemporaneous relationship between price and quantity and therefore reflects features of the underlying supply and demand curves. Demand shocks cause prices and quantities to move in the same direction, while supply shocks cause prices and quantities to move in opposite directions.

Therefore, reasonable sign restrictions can be imposed upon A such that

$$\begin{bmatrix} \epsilon_{Dt} \\ \epsilon_{St} \end{bmatrix} = \begin{bmatrix} + & + \\ - & + \end{bmatrix} \begin{bmatrix} \nu_{Pt} \\ \nu_{Qt} \end{bmatrix}$$

Therefore, a positive demand shock increases price and quantity, and a positive supply shock decreases price and increases quantity.[7]

6 This formula is used in order to keep positive and negative percent changes of the same size symmetric.

7 One reviewer worried that in response to a negative supply shock not all sectoral prices need to respond positively if there are complementarities in the production of intermediate goods; i.e., some sectoral prices could respond negatively in response to a negative supply shock. These negative price changes could then falsely lead to a classification as demand driven. Chen and Tombe (2023) deal with this issue. Their method identifies the proximate cause of a price change rather than the ultimate cause. If there is a negative supply shock, that will raise the price and lower the quantity of the good in question. But there may very well be reductions in the price of other goods that are used as an intermediate because there would be a drop in demand for those goods. That would be characterized as a negative demand shock for those goods. That isn't incorrect, since there really was a drop in demand for those goods. We also note that expectations aren't explicitly dealt with in our discussion, but that changes in firm expectations of future price changes would lead to shifts in the supply curve and therefore be (correctly) characterized as a supply driven price change today. Changes in buyer expectations of future price changes would lead to shifts in the demand curve and (correctly) be characterized as demand-driven price change.

From Chen and Tombe (2023b): "Briefly, we focus on meaningfully large deviations, or "shocks," in each item's observed prices and quantities relative to a flexible trend. If the changes in price and quantity are in the same direction (that is, both up or both down), we categorize these changes as demand shocks. Conversely, if they are in opposite directions (one is up and the other is down), we categorize these as supply shocks. Small shocks (one-fifth of all observations) are ambiguous."